11-2000

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Abstract
This study presents a spreadsheet method for analyzing the probability of completing work at all stations on linked or mechanically paced assembly lines within a given cycle time. The method can be used to determine what cycle time to set, given a desired work load at each station. Alternately, given a desired cycle time and probability of completing all work at each station within the cycle time, the method can be used to determine how much work to allocate to each station on the line. Comparisons between the output of the spreadsheet method and simulations of the same system indicate the spreadsheet method is quite accurate.

Disciplines
Operations and Supply Chain Management

ANALYZING CYCLE TIME PROBABILITIES FOR PACED OR LINKED ASSEMBLY LINES

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ABSTRACT

This study presents a spreadsheet method for analyzing the probability of completing work at all stations on linked or mechanically paced assembly lines within a given cycle time. The method can be used to determine what cycle time to set, given a desired work load at each station. Alternately, given a desired cycle time and probability of completing all work at each station within the cycle time, the method can be used to determine how much work to allocate to each station on the line. Comparisons between the output of the spreadsheet method and simulations of the same system indicate the spreadsheet method is quite accurate.

INTRODUCTION

A mechanically paced assembly line contains a set of sequential assembly stations connected by a continuous conveyor material handling system. The item being assembled is present at each station for the same fixed amount of time and at the conclusion of this cycle time, the material handling system automatically indexes each unit to the next station [1, pp. 31, 34].

One of the first decisions to make when designing a line of this type is how much work to allocate to each station given the desired cycle time and output rate from the line. If task times are variable, tasks are allocated to stations such that the average service time (ST) for each station equals the planned cycle time and the resulting service times are normally distributed, the actual ST at any station will exceed the planned cycle time 50% of the time [6]. Further, the probability that all workstations will have completed their assigned tasks before the planned cycle time elapses is 0.5^n, where n is equal to the number of stations on the line [1, p. 55]. Even for small lines of this type, the probability that all tasks are completed before the planned cycle time elapses is quite small. Consequently, the amount of work allocated to each station must be less than the planned cycle time if the planned cycle time and output rate of a mechanically paced line are to be achieved. Thus, the amount of work to allocate to each station is a critical design question.

Several prior studies have addressed the issue of how much work to allocate to each station. [3] reports that work allocations equal to 90% of planned cycle times are commonly used in practice. [9] showed that for certain classes of work element completion time distributions, the stochastic problem of assigning tasks to stations to achieve a predetermined probability of completing all tasks within the planned cycle time is equivalent to the deterministic problem of assigning tasks to stations so that the sum of the completion times is less than a fixed proportion of the cycle time. [8] developed a heuristic to minimize the probability that one or more stations would exceed the cycle time, assuming task times were normally distributed and an initial balance and cycle time were available. [5][6][7] assumed normally distributed station service times in their assembly line balancing heuristics and allocated work to a station until the average ST plus z service time standard deviations was less than or equal to the planned cycle time. The value of z determines the probability that each station completes its work before the desired cycle time elapses.

This study builds on the results of these prior studies by first developing a spreadsheet approach to examine the cycle time distribution of a linked assembly line with variable station service times. A linked line contains a set of sequential assembly stations that are connected by a continuous conveyor material handling system, but in contrast to the mechanically paced line, units are simultaneously moved to the next station on the line as soon as, but not before, all stations have finished their assembly tasks (see [4] for an industrial example of such a line).

Since the cycle time distribution of the linked line shows the probability that all stations on the line will complete their assigned assembly tasks by a given cycle time, and since the only difference between a linked and a mechanically paced line is the cycle time variability of a linked line (cycle time is fixed on a mechanically paced line), the distribution of the linked line also shows the probability that all stations on the mechanically paced line will complete their assigned assembly tasks by a given cycle time. Thus, the cycle time distribution of the linked line can be used to determine what cycle time to choose for either a paced or a linked line, given a desired work allocation to each station. Alternately, the spreadsheet approach allows a user to determine how much work to allocate to each station on a paced or linked line, given a desired cycle time and probability that all stations complete the assigned tasks within the desired cycle time. The method requires the specification of the station service time distribution but it does not require the same distribution for each station. The method is accurate and flexible, is useful to both researchers and practitioners working with the design and/or behavior of linked or paced lines, and it provides an easy-to-use teaching tool for illustrating characteristics of linked or paced assembly lines.

HUNT'S 'UNPACED BELT PRODUCTION LINE'

In 1956, Hunt [2] presented a method for determining the expected cycle time for an N station, unpaced belt production line with exponentially distributed station service times. The unpaced belt production line is the same as the linked line described in this study and will hereafter be referred to as a linked line. Since Hunt's method forms the basis for the spreadsheet methodology developed in this study, a brief review of Hunt's method is in order.

Consider an N station linked line with independent and identically distributed exponential service times at each station, a first station that is never starved, and a final station that is never blocked. Let μ be the mean service rate for each station, ST_i be the service time that actually occurs at station i in a given cycle, and Δt be the smallest measurable change in the station service time. Since station i will determine the cycle time for a given cycle if it has the longest service time of all stations, the probability that service time t occurs at station i and that service time t equals the cycle time CT is:

$$P(ST_i = t \,\&\, t = CT) = P(t \leq ST_i < t + \Delta t)* \\ P(ST_{all\ other\ stations} < t) = \int_t^{t+\Delta t} \mu e^{-\mu t}\left(1 - e^{-\mu t}\right)^{N-1} dt \qquad (1)$$

Since any of the N stations can have the longest service time in a given cycle with equal probability, the probability that CT equals t is:

$$P(CT = t) = N * P(t \leq ST_{any\,1\,station} < t + \Delta t)* \\ P(ST_{all\ other\ stations} < t) = N \int_t^{t+\Delta t} \mu e^{-\mu t}\left(1 - e^{-\mu t}\right)^{N-1} dt \qquad (2)$$

In this case, the expected cycle time E(CT) is:

$$E(CT) = N \int_0^{\infty} \mu e^{-\mu t}\left(1 - e^{-\mu t}\right)^{N-1} t\, dt \qquad (3)$$

While this method allows computation of the expected cycle time of the line given the assumptions above, it does not determine the distribution of cycle times. Thus, it gives very limited information on the expected cycle time of a linked line for distributions other than exponential, to determine how much work content to assign to a station given a desired cycle time and probability that all stations complete the assigned work before the cycle time elapses, or to examine the resulting cycle time distribution. The spreadsheet method developed in this study overcomes these limitations.

SPREADSHEET METHODOLOGY

To illustrate the spreadsheet method developed in this study, consider a three station linked line with service times at each station that are independent and normally distributed with a mean of one minute and a coefficient of variation of 0.2 minutes. Although the normal distribution is used in this example, any distribution can be used.

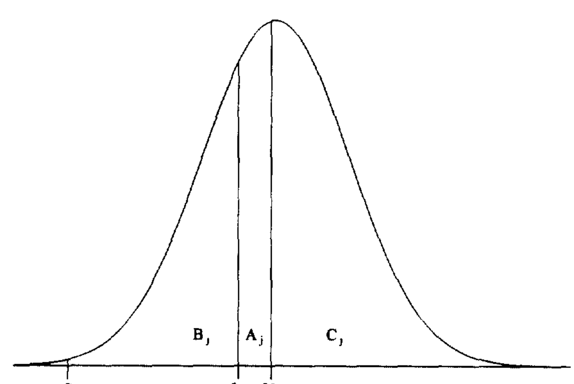
All possible service times are first divided into small, discrete time intervals. Each service time interval j has a minimum or lower bound of L_j, a maximum or upper bound of U_j, and an average service time t_j that is equal to the average of L_j and U_j (i.e. $t_j = [L_j + U_j] / 2$). Further, the upper bound of one service time interval is the lower bound of the next higher service time interval. For example, if intervals of 0.01 minutes are used, the lower bound for the first service time interval (L_1) is 0.00 minutes, the upper bound (U_1) is 0.01 minutes, and the average service time (t_1) is 0.005 minutes. Similarly, L_2 = 0.01 minutes, U_2 = 0.02 minutes, and t_2 = 0.015 minutes; L_3 = 0.02 minutes, U_3 = 0.03 minutes, and t_3 = 0.025 minutes, etc.

In order for the line to have a cycle time that falls within a particular service time interval for a given cycle, no station service time can exceed the upper bound of that interval and at least one of the three stations must have a service time that falls within that interval. Hence, the probability that the three station line will have a cycle time greater than L_j but less than or equal to U_j for a given cycle is equal to:

$$P(L_j < CT_{3\ station\ line} \leq U_j) = P(L_j < ST_{any\ 1\ station} \leq U_j)* \\ P(ST_{other\ 2\ stations} \leq L_j) + P(L_j < ST_{any\ 2\ stations} \leq U_j)* \\ P(ST_{other\ 1\ station} \leq L_j) + P(L_j < ST_{all\ 3\ stations} \leq U_j)* \\ P(ST_{no\ station} \leq L_j) \qquad (4)$$

If for a given service time interval j, the normal distribution is partitioned into three parts, A_j, B_j, and C_j (see figure 1),

Figure 1. Segmentation of normal curve



corresponding to the probabilities of having a service time greater than L_j but less than or equal to U_j, greater than zero but less than or equal to L_j, and greater than U_j, respectively, equation 4 can be written as:

$$P\left(L_j < CT_{3\,station\;line} \le U_j\right) = \binom{3}{1} A_j^1 B_j^2 + \binom{3}{2} A_j^2 B_j^1 + \binom{3}{3} A_j^3 B_j^0 = \sum_{k=1}^{3} \binom{3}{k} A_j^k B_j^{3-k} \quad (5)$$

Extending this logic to N stations gives:

$$P\left(L_j < CT_{N\,station\;line} \le U_j\right) = \sum_{k=1}^{N} \binom{N}{k} A_j^k B_j^{N-k} \quad (6)$$

Since the formula for the binomial theorem can be stated as:

$$(A+B)^N = \sum_{k=1}^{N} \binom{N}{k} A^k B^{N-k} + \binom{N}{0} A^0 B^N \quad (7)$$

and since the only difference between equations 6 and 7 is the term in equation 7 where $k = 0$, the probability of achieving a cycle time between L_j and U_j in a given cycle is:

$$P\left(L_j < CT_{N\,station\;line} \le U_j\right) = \left(A_j + B_j\right)^N - \binom{N}{0} A_j^0 B_j^N = \left(A_j + B_j\right)^N - B_j^N \quad (8)$$

The expected cycle time is found by multiplying each t_j by the probability that the cycle time for a cycle will fall within the interval containing t_j, and then summing the results. Hence, the expected cycle time is:

$$E(CT) = \sum_j t_j \left(\left(A_j + B_j\right)^N - B_j^N\right) \quad (9)$$

and the expected variance is:

$$Variance(CT) = \sum_j \left(t_j - E(CT)\right)^2 \left(\left(A_j + B_j\right)^N - B_j^N\right) \quad (10)$$

Equations 9 and 10 are easy to implement on a spreadsheet, providing the relevant service time cumulative density function formula is available.

Simulation was used to check the accuracy of the distributions computed by the spreadsheet method for a variety of service time distributions and assembly line lengths. A close match was found for all cases examined, indicating the spreadsheet method is quite accurate.

CONCLUSIONS

When properly implemented, the spreadsheet method permits: 1) the estimation of the mean and standard deviation of cycle times for the linked line, 2) the graphing of the cycle time distribution for the linked line, 3) a determination of the cycle time that should be selected for a paced line given a desired work allocation to each station, 4) a determination of how much work to allocate to each station on a linked line given a desired average cycle time, and 5) a determination of how much work to allocate to each station on a paced line given a desired cycle time and probability of completing all work within the set cycle time. A spreadsheet example and instructions on how to use it are available from the author.

Since the method uses mathematical formulas based on probability theory, the answers provided by the spreadsheet remain the same for a given system configuration and choice of service time interval. In fact, if the service time interval chosen is sufficiently small (0.01 minutes in this study), the results provided by the spreadsheet can be viewed as the long run steady state behavior of the system. Hence, the numbers generated by the spreadsheet for each system under consideration can be directly compared to determine which system is superior on a given metric. Thus, the method is quicker and easier to use than simulation (which requires multiple simulation replications, statistical analysis, etc.) for analyzing the linked and paced assembly lines described in this paper, making it a useful tool for quickly analyzing multiple 'what-if' scenarios.

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